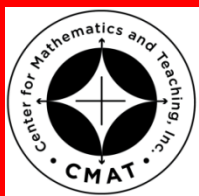


Name \_\_\_\_\_

Period \_\_\_\_\_ Date \_\_\_\_\_



**Mathlinks**

**8-11**

STUDENT PACKET

**MATHLINKS GRADE 8  
STUDENT PACKET 11  
EXPONENTS AND ROOTS**

<b>11.1</b>	<b>Squares and Square Roots</b>	<b>1</b>
	<ul style="list-style-type: none"><li>• Use numbers and pictures to understand the inverse relationship between squaring a number and finding the square root of a number.</li><li>• Approximate a square root by locating it between two consecutive integers.</li><li>• Use fractions and decimals to approximate square roots.</li><li>• Locate square roots on a number line.</li></ul>	
<b>11.2</b>	<b>Conjectures About Exponents</b>	<b>6</b>
	<ul style="list-style-type: none"><li>• Use patterns to make conjectures about multiplication and division rules for exponentials.</li><li>• Understand the meaning of zero and negative exponents.</li><li>• Use exponent definitions and rules to rewrite and simplify expressions.</li></ul>	
<b>11.3</b>	<b>Large and Small Numbers</b>	<b>18</b>
	<ul style="list-style-type: none"><li>• Read and write large and small numbers.</li><li>• Use different notations, including scientific notation, to write numbers and solve problems.</li></ul>	
<b>11.4</b>	<b>Skill Builder, Vocabulary, and Review</b>	<b>23</b>

**WORD BANK**

Word or Phrase	Definition or Explanation	Picture or Example
conjecture		
exponential notation		
factor		
product		
quotient		
radical sign		
radicand		
scientific notation		
square of a number		
square root of a number		

# SQUARES AND SQUARE ROOTS

### Summary (Ready)

We will find squares and square roots of numbers. We will approximate and compare square roots of numbers that are not perfect squares.

### Goals (Set)

- Use numbers and pictures to understand the inverse relationship between squaring a number and finding the square root of a number.
- Approximate a square root by locating it between two consecutive integers.
- Use fractions and decimals to approximate square roots.
- Locate square roots on a number line.

### Warmup (Go)

1. Draw several squares of different sizes on the grid paper below. Record the side length ( $s$ , in linear units) and area ( $A$ , in square units) for each square. One example is given.

2. Why do you think we use the word “squared” to refer to a number to the second power?

## PERFECT SQUARES

1. Complete the table.

$1^2 =$ _____	$2^2 =$ _____	$3^2 =$ _____	$4^2 =$ _____	$5^2 =$ _____
$6^2 =$ _____	$7^2 =$ _____	$8^2 =$ _____	$9^2 =$ _____	$10^2 =$ _____
$11^2 =$ _____	$12^2 =$ _____	$13^2 =$ _____	$14^2 =$ _____	$15^2 =$ _____
$16^2 =$ _____	$17^2 =$ _____	$18^2 =$ _____	$19^2 =$ _____	$20^2 =$ _____
$21^2 =$ _____	$22^2 =$ _____	$23^2 =$ _____	$24^2 =$ _____	$25^2 =$ _____

2. Based on the given example on the previous page, we write:  $5^2 =$  \_\_\_\_\_ (read “5 to the second power is \_\_\_\_\_” or “5 squared is \_\_\_\_\_”) and  $\sqrt{25} =$  \_\_\_\_\_ (read “the square root of 25 is \_\_\_\_\_”). This example illustrates the inverse relationship between squares and square roots.

Use the table above as needed, and the inverse relationship between squares and square roots, to simplify each expression.

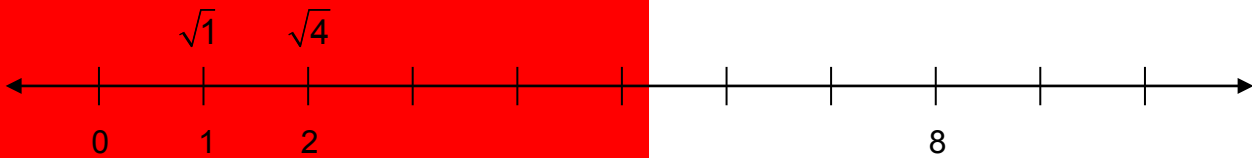
3. $\sqrt{1} =$ _____	4. $\sqrt{36} =$ _____	5. $\sqrt{81} =$ _____	6. $\sqrt{121} =$ _____
7. $\sqrt{225} =$ _____	8. $\sqrt{324} =$ _____	9. $\sqrt{400} =$ _____	10. $\sqrt{625} =$ _____

11. Alice was working with her group and said to them, “There is no square root of 13.” Explain what you think she meant by this.

## ESTIMATING SQUARE ROOTS

1. Locate the following numbers on the number line below:

$$\sqrt{0} \quad \sqrt{1} \quad \sqrt{4} \quad \sqrt{9} \quad \sqrt{16} \quad \sqrt{25} \quad \sqrt{36} \quad \sqrt{49} \quad \sqrt{64} \quad \sqrt{81} \quad \sqrt{100}$$



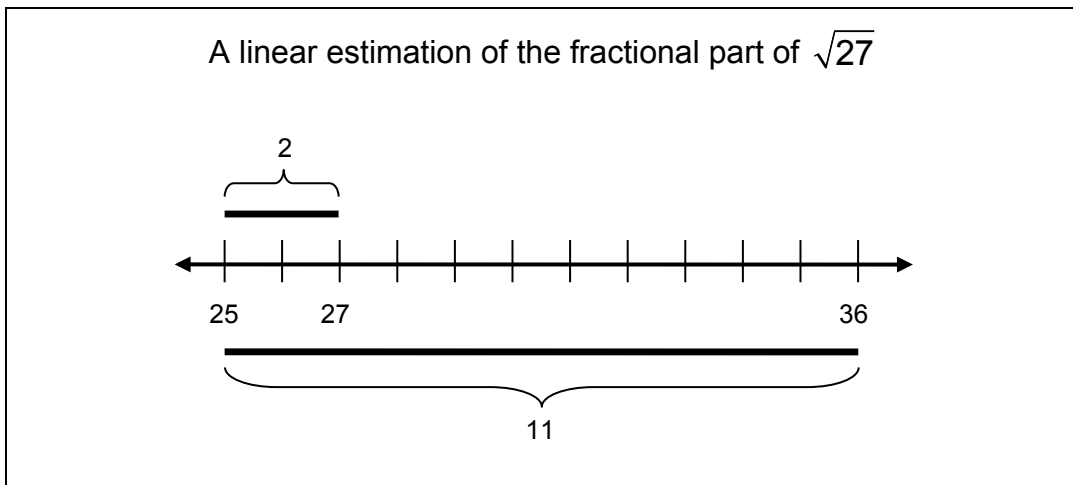
2. Use the values on the number line above to estimate the location of  $\sqrt{27}$ .

Square root form:  $\sqrt{25} < \sqrt{27} < \sqrt{36}$ , so  $\sqrt{27}$  is closer to \_\_\_\_\_.

Whole numbers:  $5 < \sqrt{27} < \text{_____}$ , so  $\sqrt{27}$  is closer to \_\_\_\_\_.

Estimate the fractional part of  $\sqrt{27}$  as a fraction.  $\frac{27 - 25}{36 - 25} = \frac{\square}{\square}$

Therefore,  $\sqrt{27}$  is about  $5 \frac{\square}{\square}$ . (Calculator check: \_\_\_\_\_)



## ESTIMATING SQUARE ROOTS (Continued)

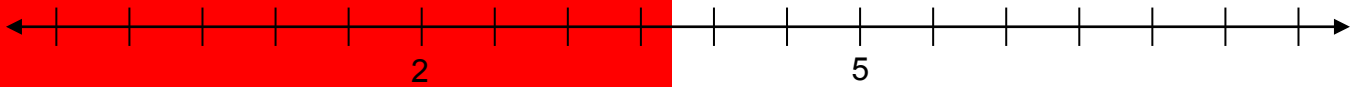
3. Determine an estimate of  $\sqrt{40}$  and then locate it on the number line below.

Square root form: \_\_\_\_\_  $< \sqrt{40} <$  \_\_\_\_\_, so  $\sqrt{40}$  is closer to \_\_\_\_\_.

Whole numbers: \_\_\_\_\_  $< \sqrt{40} <$  \_\_\_\_\_, so  $\sqrt{40}$  is closer to \_\_\_\_\_.

Estimate the fractional part of  $\sqrt{40}$  as a fraction.  $\frac{40 - 36}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$

Therefore,  $\sqrt{40}$  is about \_\_\_\_\_. (Calculator check: \_\_\_\_\_)



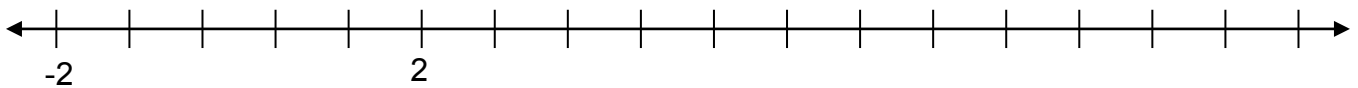
4. Determine an estimate of  $\sqrt{77}$  and then locate it on the number line below.

Square root form: \_\_\_\_\_  $< \sqrt{77} <$  \_\_\_\_\_, so  $\sqrt{77}$  is closer to \_\_\_\_\_.

Whole numbers: \_\_\_\_\_  $< \sqrt{77} <$  \_\_\_\_\_, so  $\sqrt{77}$  is closer to \_\_\_\_\_.

Estimate the fractional part of  $\sqrt{77}$  as a fraction.  $\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$

Therefore,  $\sqrt{77}$  is about \_\_\_\_\_. (Calculator check: \_\_\_\_\_)



## MORE SQUARE ROOT ESTIMATES

- Use fractions and decimals to approximate each square root.
- Use your table of squares from earlier in the lesson to help if needed.

	A Number in square root form	B Between square roots of perfect squares:	C Between 2 consecutive integers:	D About (fraction):	E About (decimal):	F Calculator check (to nearest tenth)
1.	$\sqrt{5}$	$\sqrt{4}$ and _____	2 and _____	$2 \frac{\square}{\square}$	2. _____	
2.	$\sqrt{20}$					
3.	$\sqrt{78}$					
4.	$\sqrt{220}$					
5.	$\sqrt{303}$					

For their house, Greg and Lauren bought a square rug with an area of 20 square feet.

6. If the dimensions of their front entry are 5 feet by 5 feet, will the rug fit? Explain.
7. Greg decides he would rather put the rug in front of the kitchen sink, which is a space 4 feet wide. Will the rug fit in that space? Explain.
8. Lauren thinks the rug will look great in the hallway, which is  $4\frac{1}{2}$  feet wide. Will the rug fit? Explain.
9. Greg measured the hallway again, and discovered it is actually 4 feet 4 inches wide. Will the rug fit? Explain.

## CONJECTURES ABOUT EXPONENTS

### Summary (Ready)

We will use patterns to make conjectures about rules for multiplying expressions involving exponents.

### Goals (Set)

- Use patterns to make conjectures about multiplication and division rules for exponentials.
- Understand the meaning of zero and negative exponents.
- Use exponent definitions and rules to rewrite and simplify expressions.

### Warmup (Go)

#### Exponential Notation

$b^n$  (read as “ $b$  to the power  $n$ ”) is used to express the product of  $n$  factors of  $b$ . That is,

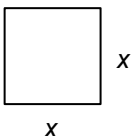
$$b^n = \underbrace{(b)(b)(b)\dots(b)}_{n \text{ factors}}.$$

The number  $b$  is the base, and the number  $n$  is the exponent. We will refer to an expression in the form  $b^n$  as in “exponential form.”

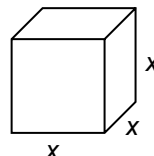
1. The expression  $3^4$  is in exponential form. The base is \_\_\_\_ and the exponent is \_\_\_\_.
2. The expression  $x^5$  is in exponential form. The base is \_\_\_\_ and the exponent is \_\_\_\_.
3. Five to the second power is written as \_\_\_\_, which is equal to \_\_\_\_  $\cdot$  \_\_\_\_ = \_\_\_\_
4.  $x$  to the third power is written as \_\_\_\_, which is equal to \_\_\_\_  $\cdot$  \_\_\_\_  $\cdot$  \_\_\_\_ = \_\_\_\_

Write an expression in exponential form to represent each of the following.

5. The area of a square with side length equal to  $x$ .



6. The volume of a cube with edge length equal to  $x$ .





## EXPONENT PRODUCT PATTERNS

Write each exponential expression as a product of factors. Then write it in exponential form ( $b^n$ ).

1.  $2^3 \cdot 2^5 = (\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad}) \cdot (\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad}) = \underline{\quad}$

2.  $x^3 \cdot x^5 = (\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad}) \cdot (\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad}) = \underline{\quad}$

3.  $3^2 \cdot 3^6 = \underline{\hspace{10cm}}$

4.  $x^2 \cdot x^6 = \underline{\hspace{10cm}}$

5. For problems 1-4, what do you notice about the bases in the original expressions compared the bases in the result?

6. For problems 1-4, what do you notice about the exponents in the original expressions compared the exponents in the result?

7.  $(8^2)^3 = (\underline{\quad})(\underline{\quad})(\underline{\quad}) = (\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad}) = \underline{\quad}$

8.  $(x^2)^3 = (\underline{\quad})(\underline{\quad})(\underline{\quad}) = (\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad}) = \underline{\quad}$


9.  $(3^4)^2 = \underline{\hspace{10cm}}$

10.  $(x^4)^2 = \underline{\hspace{10cm}}$

11. For problems 7-10, what do you notice about the exponents in the original expressions compared the exponents in the result?

## THE PRODUCT RULE FOR EXPONENTIALS

Complete problems 1-4 and write observations in the space provided about exponent relationships. Then use these observations to make a conjecture in words and symbols.



1.  $(3^2)(3^4) = (\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad}) = 3^6$  Are the bases the same?

How do the exponents in the original expression relate to the exponent in the product? In other words, how do 2 and 4 relate to 6?

2.  $(2^3)(2^4) = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}}) = 2^{\square}$

3.  $(4)(4^5) = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}}) = 4^{\square}$  (Remember that:  $4 = 4^{\square}$ )

4.  $(x^2)(x^5) = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}}) = x^{\square}$

5. Write a conjecture in words related to the multiplication in problems 1-4 above:

6. Write a conjecture in symbols:  $(x^a)(x^b) = x^{\square}$

We will call this conjecture the Product Rule for Exponentials.

# THE POWER RULE FOR EXPONENTIALS

Complete problems 1-3 and write observations in the space provided about exponent relationships. Then use these observations to make a conjecture in words and symbols.

1.  $(3^2)^4 = (3^2) \cdot (\underline{\quad}) \cdot (\underline{\quad}) \cdot (\underline{\quad})$   
 $= (\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad}) = 3^8$

How is this expression different from the expression in problem 1 on the previous page?

How do the exponents in the original expression relate to the exponent in the product? In other words, how do 2 and 4 relate to 8?

2.  $(2^3)^4 = (\underline{\quad}) \cdot (\underline{\quad}) \cdot (\underline{\quad}) \cdot (\underline{\quad})$   
 $= (\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad} \cdot \underline{\quad}) = 2^{\square}$

3.  $(x^2)^5 = (\underline{\quad}) \cdot (\underline{\quad}) \cdot (\underline{\quad}) \cdot (\underline{\quad}) \cdot (\underline{\quad})$   
 $= (\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad})(\underline{\quad} \cdot \underline{\quad}) = x^{\square}$

4. Write a conjecture in words related to the multiplication in problems 1-3 above:

5. Write a conjecture in symbols:  $(x^a)^b = x^{\square}$

We will call this conjecture the Power Rule for Exponentials.

## EXPONENT PRACTICE 1

Write each expression in exponential form ( $b^n$ ). Circle the rule used (Product Rule or Power Rule for Exponentials.)

1. $5^6 \cdot 5^3 = 5^{\square} = 5^{\square}$ Product Rule      Power Rule	2. $(5^6)^3 = 5^{\square} = 5^{\square}$ Product Rule      Power Rule
3. $(7^2)(7^4)$ Product Rule      Power Rule	4. $(7^2)^4$ Product Rule      Power Rule
5. $19^8 \cdot 19^6$ Product Rule      Power Rule	6. $(19^8)^6$ Product Rule      Power Rule
7. $10 \cdot 10^4$ Product Rule      Power Rule	8. $(10^1)^4$ Product Rule      Power Rule
9. $10^4 \cdot 10$ Product Rule      Power Rule	10. $(10^4)^1$ Product Rule      Power Rule
11. $y^{10} \cdot y^{10}$ Product Rule      Power Rule	12. $(y^{10})^{10}$ Product Rule      Power Rule
13. $(x^5)(x^6)$ Product Rule      Power Rule	14. $(x^5)^6$ Product Rule      Power Rule
15. $x \cdot x^8$ Product Rule      Power Rule	16. $(x^1)^8$ Product Rule      Power Rule

## EXPONENT PRACTICE 1 (Continued)

Compute. Results should contain no exponents.

17. $\left(\frac{1}{4}\right)^2$	18. $\left(\frac{2}{3}\right)^3$
19. $\left(\frac{1}{4}\right)^2 \cdot \left(\frac{1}{4}\right)^3$	20. $\left(\left(\frac{2}{3}\right)^3\right)^2$
21. $3^2 + 2^3$	22. $(3^2)(2^3)$
23. $2^3 + 2^5$	24. $2^{(3+5)}$

25. Why does the product rule NOT apply to problem 22?

26. Why does the power rule NOT apply to problem 24?

27. Write three different expressions equivalent to  $2^8$  that include exponents. (Computing  $2^8$  is not necessary.)

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28. If  $\left(\frac{1}{4}\right)^3 = \left(\frac{1}{2}\right)^n$ , what is the value of  $n$ ?

For each equation below, find all integer values for  $x$  that make the equation true.

29. $x^2 = 16$	30. $x^2 = 144$
31. $x^3 = 8$	32. $x^3 = 1$

## PAPER FOLDING EXPERIMENT

1. Record the results from folding a piece of paper.

Number of Folds	Number of Sections	Number of Sections as 2 to a Power
1	2	$2^{\square}$
2		
3		
4		
5		
6		

2. The paper folding experiment suggests that  $2^0 = \underline{\hspace{2cm}}$

3. Rule:  $x^0 = \underline{\hspace{2cm}}$ ,  $x \neq 0$ .

## EXPONENTIAL QUOTIENT PATTERNS

Complete the table. Following patterns down each column may be helpful.

	I. Expression	II. Expanded Form	III. Power of 10 (fractions okay)	IV. Power of 10 (no fractions)	V. Value (fractions okay)
1.	$\frac{10^3}{10^0}$			$10^3$	1000
2.	$\frac{10^3}{10^1}$	$\frac{10 \cdot 10 \cdot 10}{10}$			
3.	$\frac{10^3}{10^2}$		$10^1$	$10^1$	
4.		$\frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10}$			
5.	$\frac{10^3}{10^4}$		$\frac{1}{10^1}$		$\frac{1}{10}$
6.	$\frac{10^3}{10^5}$			$10^{-2}$	

7. The expressions in column I are quotients with the same bases. The expressions in column IV are in exponential form. What fact about the exponents in column I determine when the exponents in column IV will be...
- Positive?
  - Negative?
  - Zero?
8. How is  $10^{-2}$  related to  $10^2$ ?
9. Do any of the expressions above with negative exponents result in a negative value?

## EXPONENTIAL QUOTIENT PATTERNS (Continued)

Complete the table. Following patterns down each column may be helpful.

	I. Expression	II. Expanded Form	III. Power of 10 (fractions okay)	IV. Power of 10 (no fractions)	V. Value (fractions okay)
10.	$\frac{3^1}{3^0}$				
11.			$3^0$	$3^0$	
12.	$\frac{3^1}{3^2}$				$\frac{1}{3}$
13.	$\frac{3^1}{3^3}$		$\frac{1}{3^2}$		
14.		$\frac{3}{3 \cdot 3 \cdot 3 \cdot 3}$			$\frac{1}{27}$
15.	$\frac{3^1}{3^5}$			$3^{-4}$	

16. Patti thinks that a base number to a negative power must result in a negative value. Is Patti correct? Explain.

17. How is  $3^{-1}$  related to  $3^1$ ?

18. Make conjectures: If  $x \neq 0$  and  $a > 0$ , then

$$x^0 = \underline{\hspace{2cm}}$$

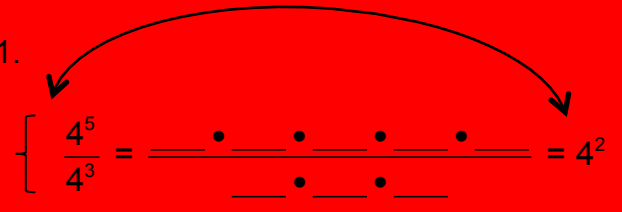
$$x^{-1} = \underline{\hspace{2cm}}$$

$$x^{-a} = \underline{\hspace{2cm}}$$



## THE QUOTIENT RULE FOR EXPONENTIALS

Complete problems 1-4 and write observations in the space provided about exponent relationships. Then use these observations to make a conjecture in words and symbols.

<p>1. </p>	<p>Are the bases the same? _____</p>
<p>2. <math>\frac{4^3}{4^5} = \frac{\text{---} \cdot \text{---} \cdot \text{---}}{\text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---}} = \frac{1}{4^{\square}} = 4^{\square}</math></p>	
<p>3. <math>\frac{x^5}{x^2} = \frac{\text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---}}{\text{---} \cdot \text{---}} = x^{\square}</math></p>	
<p>4. <math>\frac{x^2}{x^5} = \frac{\text{---} \cdot \text{---}}{\text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---} \cdot \text{---}} = \frac{1}{x^{\square}} = x^{\square}</math></p>	
<p>5. Write a conjecture in words related to the division in problems 1-4 above:</p>   <p style="text-align: center;">If <math>x \neq 0</math>, then <math>\frac{x^a}{x^b} = x^{\square}</math></p>	
<p>We will call this conjecture the <u>Quotient Rule for Exponentials</u>.</p>	

## EXPONENT PRACTICE 2

1. A number to a negative power is the same as the \_\_\_\_\_ of the number to a positive power. In other words:

$$b^{-n} = \frac{1}{b^n} = \left(\frac{1}{b}\right)^n$$

2. Using the quotient rule, write  $\frac{5^4}{5^8}$  as 5 to a single power. \_\_\_\_\_

Now write this expression with a positive exponent. \_\_\_\_\_

Compute.

3. $\frac{3^4}{3^4}$	4. $3^4 \cdot \frac{1}{3^4}$	5. $3^4 \cdot 3^{-4}$
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Compute. Fractions are okay.

6. $\frac{(5^2)^2}{5^3}$	7. $\frac{5^3 \cdot 5^2}{(5^3)^2}$	8. $\frac{5^3 \cdot 5^2}{5^4 \cdot 5^2}$	9. $\frac{(5^2)^3}{5^6}$
10. $2^{-3} \cdot 2^5$	11. $2^3 \cdot 2^{-5}$	12. $\frac{2^2 \cdot 2^{-3}}{2^3}$	13. $\frac{2^{-3} \cdot 2^{-3}}{2^{-2}}$

## EXPONENT PRACTICE 2 (Continued)

Compute.

14. $(-2)^2$	15. $(-2)^3$	16. $(-2)^4$	17. $(-2)^5$
18. $(-2)^{-2}$	19. $(-2)^{-3}$	20. $(-2)^{-4}$	21. $(-2)^{-5}$

22. Karin thinks that a negative number to a negative power must be negative. Is she correct? Explain.

Write each expression in exponential form  $b^n$ , where  $b > 1$ . The exponent need not be positive.

23. $\frac{6^5}{6}$	24. $\frac{(5^2)^3}{5^8}$	25. $\frac{2 \cdot 3^{15}}{6 \cdot 3^{17}}$	26. (hint: negative exponent → reciprocal) $\left(\frac{1}{2}\right)^{-3}$
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Write each expression in exponential form. The base ( $b$ ) is given.

27. $\frac{9^6}{9^4} = 9^{\square}$	28. $\frac{9^4}{9^6} = 9^{\square}$	29. $\frac{9^6}{9^4} = 3^{\square}$	30. $\frac{9^4}{9^6} = 3^{\square}$
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# LARGE AND SMALL NUMBERS

### Summary (Ready)

We will write large and small numbers using a variety of notations, including scientific notation. We will solve problems involving large numbers.

### Goals (Set)

- Read and write large and small numbers.
- Use different notations, including scientific notation, to write numbers and solve problems.

### Warmup (Go)

1. Fill in the place value chart below and locate the decimal point.

trillions							hundred thousands				hundreds			ones			hundredths		
-----------	--	--	--	--	--	--	-------------------	--	--	--	----------	--	--	------	--	--	------------	--	--

Write in words:

2. 1,020,300,004,500 \_\_\_\_\_  
 \_\_\_\_\_

3. 0.078 \_\_\_\_\_

4. 0.0006 \_\_\_\_\_

Round to two significant digits.

5. 9,564 \_\_\_\_\_

6. 5.0309 \_\_\_\_\_

Round to one significant digit.

7. 20,345,678 \_\_\_\_\_

8. 0.000494 \_\_\_\_\_

## LARGE NUMBERS

Scientific notation is a system that is useful for writing large numbers. In scientific notation, a number is written as a decimal that is greater than or equal to 1 and less than 10, multiplied by a power of 10.

	Standard notation	Product of a number between 1 and 10, and a multiple of 10	Scientific notation
Ex.	5,200	$5.2 \times 1,000$	$5.2 \times 10^3$
1.	479,000,000	$4.79 \times 100,000,000$	$4.79 \times 10^{\square}$
2.	2,000	$2 \times \square$	$2 \times 10^{\square}$
3.	68,000,000	$\square \times \square$	$\square \times 10^{\square}$
4.		$4.58 \times 10,000$	
5.		$2.6 \times 1,000,000$	
6.			$5.1 \times 10^4$
7.			$3.07 \times 10^5$

8. Explain why  $41.2 \times 10^5$  is NOT in scientific notation. Then rewrite it in scientific notation.

## SMALL NUMBERS

Scientific notation is also useful for writing very small numbers. In scientific notation, a number is written as a decimal greater than or equal to 1 and less than 10, multiplied by a power of 10.

	Standard notation	Product of a number between 1 and 10, and a multiple of 10		Scientific notation
Ex.	0.007	$7 \times 0.001$	$7 \times \frac{1}{1000}$	$7 \times 10^{-3}$
1.	0.023	$2.3 \times 0.01$	$2.3 \times \square$	$2.3 \times 10^{\square}$
2.	0.000459	$4.59 \times \square$	$4.59 \times \square$	$4.59 \times 10^{\square}$
3.	0.0061			$\square \times 10^{\square}$
4.		$7.58 \times 0.0001$		
5.			$6.2 \times \frac{1}{1000}$	
6.				$9.1 \times 10^{-4}$
7.				$8.03 \times 10^{-5}$

8. Explain why  $0.21 \times 10^{-5}$  is NOT in scientific notation. Then rewrite it in scientific notation.

## PRACTICE WITH SCIENTIFIC NOTATION

Determine whether each number is in scientific notation. If NOT, write it in scientific notation.

1. $89.2 \times 10^8$	2. $145 \times 10^5$
3. $0.23 \times 10^{-6}$	4. $4.55 \times 10^{-10}$
5. $0.043 \times 10^{-4}$	6. $2.4 \times 10^5$

7. Circle the greater number. The number you circled is how many times as large as the smaller number?

$$3.5 \times 10^3 \qquad 7 \times 10^5$$

Use symbols  $<$ ,  $=$ , or  $>$  to compare each pair of numbers.

8. $1.65 \times 10^5$ <input type="text"/> $4.51 \times 10^3$	9. $3.2 \times 10^{-6}$ <input type="text"/> $3.2 \times 10^6$
10. $8 \times 10^9$ <input type="text"/> $8.1 \times 10^9$	11. $5.2 \times 10^7$ <input type="text"/> $4.23 \times 10^6$
12. $6.41 \times 10^{-5}$ <input type="text"/> $5.73 \times 10^{-6}$	13. $1.1119 \times 10^{-4}$ <input type="text"/> $1.1 \times 10^{-5}$

14. Jorge multiplied 60 million by 4 billion on his calculator. The calculator display showed:

2.4 E17

Explain to him what you think this means.

# WORLD POPULATION

1. Complete the table.

Country	Population			US population is how many times larger? (estimate)
	rounded to the nearest thousand	rounded to one significant digit	written as a whole number times a power of 10	
United States	315,250,000	300,000,000	$3 \times 10^8$	
United Kingdom	63,182,000			
South Korea	50,004,000			
Uzbekistan	29,559,000			
Syria	21,949,000			
Somalia	9,797,000			
Paraguay	6,337,000			
Ireland	4,588,000			
Iceland	320,000			

Source: Wikipedia, February, 2013. Figures are based on the most recent estimate or projections by the national census authority where available and rounded off.

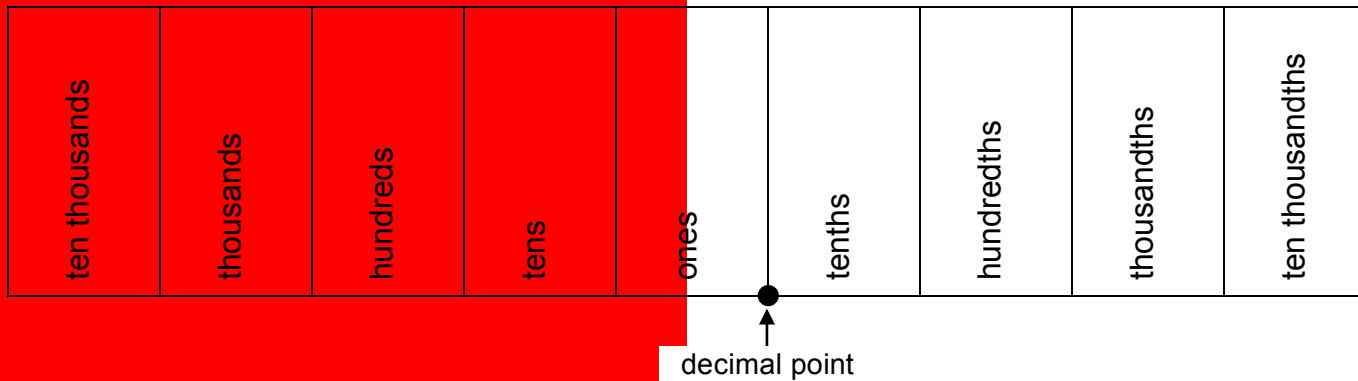
2. The world population is the total number of living humans on Earth. A 2013 estimate by the United States Census Bureau (USCB) is 7.063 billion people. The two most populous countries in the world are India and China. Round the population numbers appropriately to help you estimate what fraction of the total world population lives in these countries.

India (1,210,193,000)	China (1,354,040,000).
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# SKILL BUILDERS, VOCABULARY, AND REVIEW

## SKILL BUILDER 1



1. What does the 7 represent in 84.273? \_\_\_\_\_
2. What does the 3 represent in 84.273? \_\_\_\_\_
3. Write 30.194 in words. \_\_\_\_\_
4. Circle the digit in the tenths place.                      3 0 . 1 9 4
5. Circle the digit in the tens place.                              3 0 . 1 9 4
6. Round 457.568 to the nearest tenth.                      \_\_\_\_\_
7. Round 79.302 to the nearest hundredth.                      \_\_\_\_\_

Solve mentally.

8.  $24 - b = 8$ ,  $b =$  \_\_\_\_\_
9.  $-3(p + 7) = -36$ ,  $p =$  \_\_\_\_\_

Solve.

10. $3(x + 6) - 12 = 3x - 6$	11. $\frac{1}{6} = 2x - \frac{1}{5}$
------------------------------	--------------------------------------

Solve as indicated.

12. Circumference of a circle:  
 $C = \pi d$ , solve for  $d$
13. Volume of a rectangular prism;  
 $V = \ell wh$ , solve for  $h$

**SKILL BUILDER 2**

Compute.

1. $-7 - (-9)$	2. $9 - (-7)$	3. $-3(-7 - 2)$	4. $5 - (-6 + 12)$
5. $\frac{-8 - 16}{-2}$	6. $\frac{-4(-1 + 9)}{-8}$	7. $\frac{5 - 35}{-6 - 14}$	8. $\frac{4}{-7 + 15} + \frac{-3 - 3}{22 + (-14)}$

Solve.

9. $-8x + (5x - 7) = -(4x - 2) - 9$	10. $-\frac{1}{2}(x + 10) = \frac{3}{4}(x - 8)$
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Solve each system using the substitution method or the elimination method.

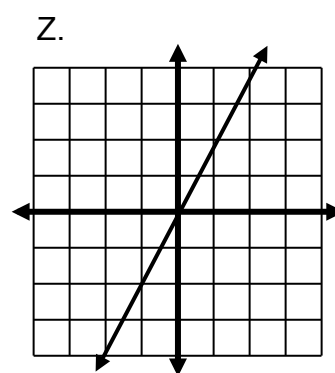
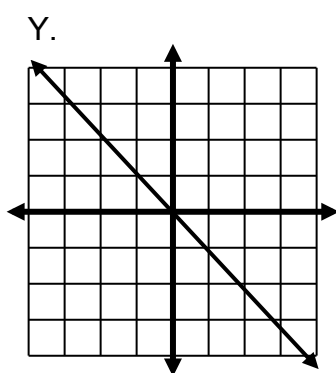
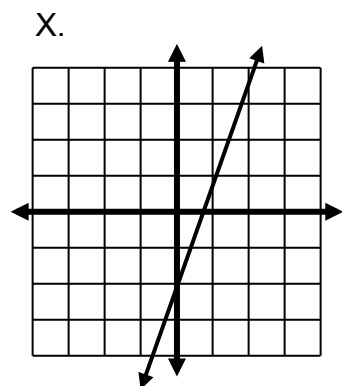
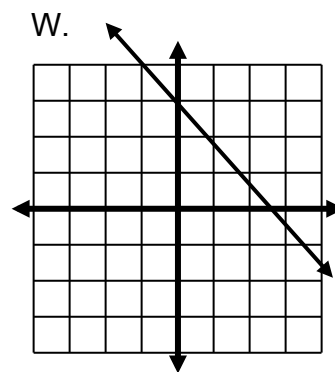
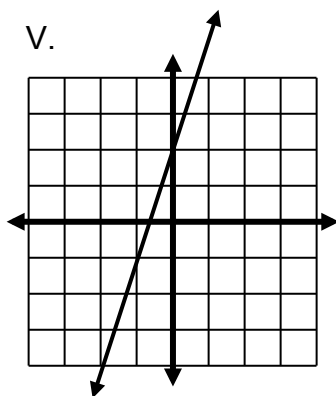
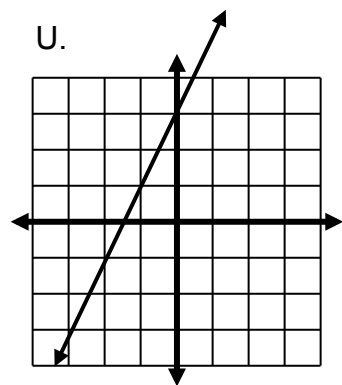
11. $\begin{cases} y + 3x = 1 \\ 2x - 4 = y \end{cases}$	12. $\begin{cases} \frac{1}{3}x - 2y = -3 \\ y = 3 \end{cases}$
--	---

### SKILL BUILDER 3

- Match each equation (1-6) to one set of ordered pairs (a-f) and also to one graph (U-Z).
- **Circle** the equation(s) of the line(s) with the greatest slope.
- **Box** the equation(s) of the line(s) with the least slope.

Equation	Ordered pairs	Graph
1. $y = 2x$		
2. $y = 2x + 3$		
3. $y = 3x + 2$		
4. $y = 3x - 2$		
5. $y = -x + 3$		
6. $y = -x$		

a. (0, 0) (1, -1) (-1, 1)	b. (0, 0) (1, 2) (-1, -2)
c. (0, 3) (1, 5) (-1, 1)	d. (0, 3) (1, 2) (-1, 4)
e. (0, 2) (1, 5) (-1, -1)	f. (0, -2) (1, 1) (-1, -5)



### SKILL BUILDER 4

A hundred 8<sup>th</sup> grade boys were asked the following questions and frequency tables were constructed based on the data.

- Is your shoe size a size 10 or larger?
- Are you taller than 5 feet?

	shoe size < 10	shoe size ≥ 10	total
shorter than 5 feet tall	93%	7%	100%
taller than 5 feet tall	50%	50%	100%

	shoe size < 10	shoe size ≥ 10
shorter than 5 feet tall	81%	25%
taller than 5 feet tall	19%	75%
total	100%	100%

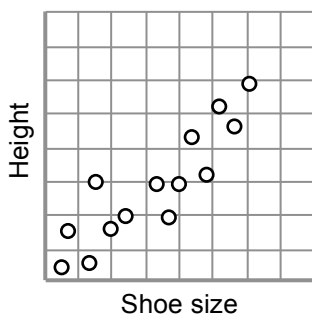
Use the data above. Draw lines to match the percent with the correct statement.

- 93%      A. of students who wear a shoe size less than 10 are less than 5 feet tall.
- 75%      B. of students who are less than 5 feet tall wear a shoe size less than 10.
- 81%      C. of students who are less than 5 feet tall wear shoe size 10 or larger.
- 7%        D. of students who have a shoe size 10 or larger are more than 5 feet tall.

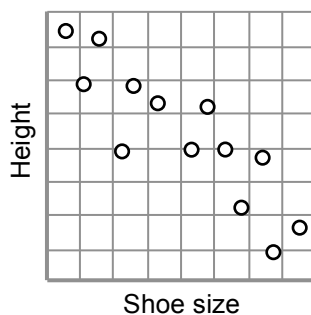
5. Describe the association between height and shoe size. \_\_\_\_\_

6. If numerical data was collected about the height and shoe size of these same 8<sup>th</sup> graders and graphed as a scatter plot, which scatter plot might best represent the data? Explain.

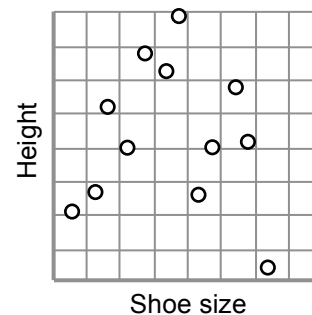
Height vs Shoe Size



Height vs Shoe Size

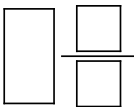


Height vs Shoe Size



### SKILL BUILDER 5

- Use fractions and decimals to approximate each square root.
- Use your table of squares from page 2 to help if needed.
- Then label the tick marks on the number line below and graph the approximate locations of all the numbers in problems 1 and 2. The distance from one tick mark to the next need not be 1 unit of length.

	A Number	B Between square roots of perfect squares:	C Between 2 consecutive integers:	D About (fraction):	E About (decimal):	F Calculator check (to nearest tenth)
1.	$\sqrt{17}$	_____ and _____	_____ and _____		_____ . _____	
2.	$\sqrt{38}$					



- You have a square plot of land of 800 sq. feet that you want to use as a vegetable garden. Your neighbor offers you a tool shed that is 6 ft. x 30 ft.

  - What amount of land (area) would this shed need?
  - Will this shed fit on your land? Explain.
  - How much land is left for the vegetable garden?

- Find the volume of the right rectangular prism with the following dimensions. The length of the prism is 20 cm. The width is half the length. The height is 5 less than the width.

$l = \underline{\hspace{2cm}}$      $w = \underline{\hspace{2cm}}$      $h = \underline{\hspace{2cm}}$

Volume of the prism =  $\underline{\hspace{4cm}}$

**SKILL BUILDER 6**

Write each expression in exponential, form  $(b^n)$ .

1. $7^8 \cdot 7^9$	2. $(12^4)^3$	3. $x^5 \cdot x^7$
4. $(x^5)^5$	5. $13^{-7} \cdot 13^{10}$	6. $(x^{-3})^5$
7. $\frac{(6^4)^3}{6^6 \cdot 6^5}$	8. $\frac{x^3 \cdot x^4}{x^9}, (x \neq 0)$	9. $\frac{(x^2)^3}{x^7}, (x \neq 0)$

10. Compute:  $2^3 + 2^4$ .

Can this value be written as 2 raised to a whole number power? Use some examples to justify your answer.

Compute. Fractions are okay.

11. $4^2$	12. $(-4)^2$	13. $4^{-2}$	14. $(-4)^{-2}$
-----------	--------------	--------------	-----------------

15. Write  $x^{-4}$  in three different but equivalent ways that include ,s.

16. Write two expressions equal to 32 that are in the form  $2^m \cdot 2^n$ , where  $m$  and  $n$  are integers.

## SKILL BUILDER 7

**ERROR ALERT!** Be careful with ,ial expressions that include a minus sign.

The minus sign is part of the base

$$(-4)^2 = (-4)(-4) = 16.$$

The minus sign is not part of the base

$$-4^2 = -(4)(4) = -16.$$

1. Here are four expressions that contain exponents:  $3^2$ ,  $(-3)^2$ ,  $-3^2$ ,  $0 - 3^2$

Circle the expressions that are equal to 9.

Show work to justify:

Box the expressions that are equal to -9.

Show work to justify:

Compute.

2. $\frac{5}{5^4}$	3. $\frac{(8^2)^4}{8^9}$	4. $2^6 + 6^2$
5. $6 - 5^2$	6. $(-8)^2$	7. $-7^2$
8. $\left(\frac{1}{4}\right)^{-2}$	9. $\left(\frac{3}{4}\right)^{-2}$	10. $\left(\frac{1}{4}\right)^{-3}$

11. Write numerical examples for the following statements. Show work to justify the answers.

a. A negative number to a negative power that has a positive value.	b. A negative number to a negative power that has a negative value.
---	---

**SKILL BUILDER 8**

Determine whether each number is in scientific notation. If NOT, write it in scientific notation.

1.  $346 \times 10^5$

2.  $0.22 \times 10^{-8}$

Use symbols  $<$ ,  $=$ , or  $>$  to compare each pair of numbers. Show work.

3.  $1.5 \times 10^8$    $451 \times 10^5$

4.  $0.123 \times 10^4$    $3.2 \times 10^{-6}$

5. Abel multiplied two large numbers on his calculator and got:

Explain to him what his calculator is doing, and the number this represents.

6. Consider the numbers  $50 \times 10^{-8}$  and  $0.5 \times 10^{-4}$ .

a. Write both numbers as a single digit times a power of 10, and circle the greater number.

b. How many times as large is the number you circled compared to the smaller one?

7. Dr. Jerry Buss purchased the Los Angeles Lakers basketball team in 1979 for approximately \$67.5 million. At his death in 2013, the team was reportedly worth about \$1 billion.

a. Write each dollar amount as a number rounded to two significant digits.

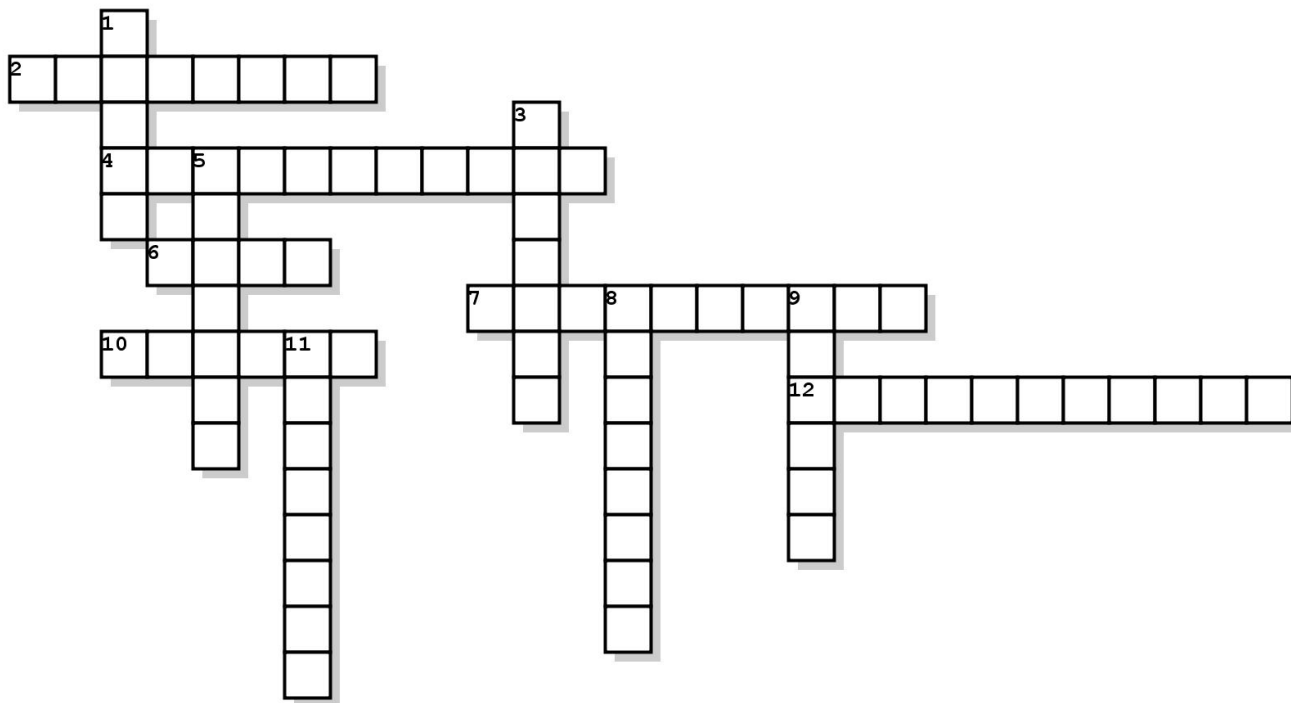
b. Write each value using scientific notation.

c. The value of the Lakers at Dr. Buss' death is about how many times as large as the value when he purchased the team?



## FOCUS ON VOCABULARY

Use vocabulary in the word bank and throughout this packet to complete the crossword.



<u>Across</u>	<u>Down</u>
2 $\frac{3^5}{3^2} = 3^{5-2}$ is an example of the _____ rule for exponentials.	1 $(2^3)^4 = 2^{3 \cdot 4}$ is an example of the _____ rule for exponentials.
4 $5^3 4^2$ is in _____ notation.	3     the name of this symbol: $\sqrt{\quad}$
6     9 is the square _____ of 81.	5 $5^4 5^2 = 5^{4+2}$ is an example of the _____ rule for exponentials.
7 $5.2 \times 10^{-2}$ is in _____ notation.	8     5.5 is an _____ for $\sqrt{30}$
10    25 is the _____ of (-5)	9     4 is a _____ of 40
12    6, 7, and 8 are _____ numbers.	11    The name of 30 in 8 down is _____.

**SELECTED RESPONSE**

Show your work on a separate sheet of paper and choose the best answer(s).

---

1. Choose all expressions equivalent to  $2^{12}$ .

A.  $(2^3)^4$

B.  $2^6 \cdot 2^2$

C.  $2^{-1} \cdot 2^{13}$

D.  $2^4 \cdot 2^8$ 

---

2. Choose all expressions equivalent to  $(x^5)^2$ .

A.  $x^7$

B.  $x^{10}$

C.  $2x^5$

D.  $\frac{x^{20}}{x^{10}}$ 

---

3. Choose all expressions equivalent to  $10 \cdot 10^6$ .

A.  $10^7$

B.  $10^6$

C.  $(10^2)^3$

D.  $10^{-5}$ 

---

4. Choose all expressions equivalent to  $\frac{y^7}{y^2}$ .

A.  $y^5$

B.  $y^{-5}$

C.  $y^9$

D.  $\frac{y^{-2}}{y^{-7}}$ 

---

5. Choose all symbols that make  $\frac{(7^4)^2}{7^8} \square 7^0$  a true statement.

A.  $<$

B.  $>$

C.  $=$

D.  $\geq$ 

---

6. Express  $5.50 \times 10^{-6}$  in standard notation.

A. 550,000

B. 5,500,000

C. 0.00000055

D. 0.0000055

---

7. Express 17,000,000 in scientific notation.

A.  $1.7 \times 10^7$

B.  $1.7 \times 10^6$

C.  $1.7 \times 10^5$

D.  $17 \times 10^6$ 

---

8. Which of these numbers has a value between 11 and 12?

A.  $\sqrt{107}$

B.  $\sqrt{120}$

C.  $\sqrt{136}$

D.  $\sqrt{145}$ 

---

9. The square root of 91 is between

A. 8 and 9

B. 9 and 10

C. 10 and 11

D. 11 and 12

---

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

### 11.1 Squares and Square Roots

Find the two consecutive integers such that the square root lies between them.

1.  $\sqrt{59}$

2.  $\sqrt{136}$

Use fractions and decimals to express each square root.

3.  $\sqrt{39}$

4.  $\sqrt{105}$

### 11.2 Conjectures About Exponents

Compare. Use  $>$ ,  $<$ , or  $=$  to complete each statement.

5.  $(7^2)(7^3) \square (7^3)^2$

6.  $\frac{9^8}{9^5} \square (9^8)^5$

Write in exponential form.

7.  $x^5 \cdot x^7$

8.  $(x^5)^7$

9.  $\frac{(5^4)^3}{5^7}$

10.  $\frac{(5^4)^2}{5^3 \cdot 5^7}$

11. Complete the equation  $\frac{1}{8^2} = 8^{\square}$ .

12. Compute  $\left(\frac{1}{5}\right)^{-2}$

### 11.3 Large and Small Numbers

13. Write the number 0.000638 in scientific notation.

14. Write the number  $5.79 \times 10^7$  in standard notation.

## HOME-SCHOOL CONNECTION

Here are some questions from this week's lessons to review with your young mathematician.

1. Approximate  $\sqrt{70}$  in the following 3 ways:
  - a. Listing the two consecutive whole numbers between which it lies.
  - b. A mixed number (whole number plus fraction).
  - c. A decimal number (whole number plus decimal fraction).
2. Express the number  $\frac{8^4 \cdot 8^6}{(8^3)^5}$ 
  - a. As a power of 8 (in the form  $8^n$ )
  - b. As a power of 2 (in the form  $2^n$ )
3. Give an example a numerical expression in exponential form,  $b^n$ , where  $n$  is a negative number but the value of the expression is positive.
4. Give an example of a negative number I exponential form  $b^n$ , where  $n > 0$ .
5. Explain why  $0.2 \times 10^{-4}$  is not in scientific notation, and then write it in scientific notation.

Parent (or Guardian) Signature \_\_\_\_\_

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# COMMON CORE STATE STANDARDS – MATHEMATICS

## STANDARDS FOR MATHEMATICAL CONTENT

- 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). *For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
- 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .*
- 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. ~~Know that  $\sqrt{2}$  is irrational.~~
- 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.*
- 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

## STANDARDS FOR MATHEMATICAL PRACTICE

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.



9 7 8 1 6 1 4 4 5 2 1 9 5

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